Coherent Thermoelectric Effects in Mesoscopic Andreev Interferometers

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We investigate thermoelectric transport through Andreev interferometers. We show that the ratio of the thermal and the charge conductance exhibits large oscillations with the phase difference ϕ between the two superconducting contacts, and that the Wiedemann-Franz law holds only when $\phi = \pi$. A large average thermopower furthermore emerges whenever there is an asymmetry in the dwell times to reach the superconducting contacts. When this is the case, the thermopower is odd in ϕ. In contrast, when the average times to reach either superconducting contact are the same, the average thermopower is zero, however mesoscopic effects (analogous to universal conductance fluctuations) lead to a sample-dependent thermopower which is systematically even in ϕ.

Introduction. The processing of information unavoidably generates heat [1]. Conventional microelectronics use electric potentials to switch currents on and off. At the nanoscale, however, this becomes energetically prohibitive and generates an amount of heat that is hard to dissipate. As new architectures are explored for quantum communication and computing, the question of dissipating heat is again of central importance. In current prototypes for quantum information processors, one of the slowest steps is cooling down the qubits in between computations [2], thus it is crucial to understand heat flows at the nanoscale and sub-Kelvin temperatures where quantum coherent effects are ubiquitous.

While quantum interference effects occur in all mesoscopic systems [3], many of them are hugely magnified by Andreev reflection in normal-metallic/superconducting nanostructures [4, 5, 6, 7]. Experiments on Andreev interferometers – metallic constrictions contacted to two superconducting terminals with a phase difference, ϕ – have revealed thermoelectric properties that are strongly affected by these magnified interference effects [5, 6, 8, 9, 10, 11, 12, 13]. The device properties can be probed (and controlled) by varying ϕ with an applied magnetic flux or a supercurrent. For instance, the charge, G, and thermal, Ξ, conductances and the thermopower, S, oscillate periodically with ϕ. The salient observations are that (i) the amplitude of the conductance oscillations can largely exceeds e^2/h and is typically larger in samples with larger average conductance, (ii) G has its maxima where Ξ has its minima and vice-versa in violation of the Wiedemann-Franz law, (iii) S is significantly larger than in normal metals in absence of superconductivity, (iv) S is either even or odd in ϕ, depending on the interferometer geometry, and (v) S exhibits oscillations of maximal amplitude at an intermediate temperature. Despite extensive theoretical investigations [14, 15, 16, 17, 18, 19, 20, 21], a unified theoretical picture of these observations is still lacking. In particular, the existing scenarios for an odd thermopower are always associated with a temperature

gradient between the two contacts to the superconducting terminals [18, 20, 21]. It is thus unclear whether these theories can capture the recently observed odd thermopower [13] in the geometry of Fig. 1d, where this gradient most likely vanishes.

Motivated by these experimental findings, we investigate experimentally relevant models of Andreev interferometers where ideal metallic leads carrying $N_i \gg 1$ modes are connected at either chaotic ballistic or disordered quantum dots with no spatial symmetry. The dots are contacted to two s-wave superconductors with order parameters $\Delta e^{i\phi_i}$, each carrying $N_{\mathrm{S}i}$ channels, $i = \mathrm{L}, \mathrm{R}$. Physical properties depend only on the phase difference $\phi_\mathrm{L} - \phi_\mathrm{R}$, so we set $\phi_\mathrm{L} = \phi/2$ and $\phi_\mathrm{R} = -\phi/2$ with $\Delta \in \mathcal{R}$. We take the superconductors to be islands through which no current flows on time average in steady-state, as appropriate to the experiments of Refs. [5, 9, 10, 11, 12, 13]. The models are sketched in Fig. 1 and are devised to have the same topology as the house (a and b) and parallelogram (c) interferometers of Refs. [10, 11], and (d) the interferometer of Ref. [13] which we call *hot-middle*. These models differ by the absence (a) or presence (b,c,d) of correlation between the action phase a quasiparticle accumulates and the superconducting phase it acquires at Andreev reflections. This correlation is key to understanding the large thermopowers induced by the presence of superconductivity, because it breaks particle-hole symmetry.

The theory we are about to present gives a unified picture of thermoelectric transport through mesoscopic Andreev interferometers. Extrapolated to diffusive systems, it exhibits all the main experimental observations listed above. For the asymmetric house (model b), parallelogram and hot-middle interferometers, it predicts that on average the thermopower will be an odd oscillatory function of ϕ, even in the absence of a temperature difference between the two superconducting contacts. In contrast, for the symmetric-house interferometer it predicts that the average thermopower is zero, but that mesoscopic
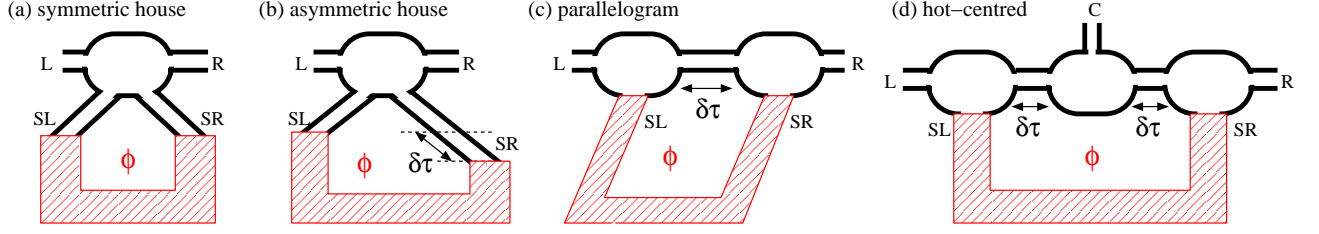
Figure 1: The four Andreev interferometers considered in this letter. (a)-(b) Single-cavity, two-terminal models, topologically equivalent to the house interferometer of Ref. [10]; the symmetry between the leads to the superconducting contacts is broken in model (b). (c) Double-dot model, topologically equivalent to the parallelogram interferometer of Ref. [10]. (d) Triple-dot model topologically equivalent to the hot-middle interferometer of Ref. [13], with a hotter lead M.

fluctuations render the thermopower random in sign, but systematically even in ϕ.

Thermoelectric transport. The linear response expression for charge, I_i, and heat, J_i, currents in lead i [17] (summation over doubly-occurring indices is assumed) is

$$\begin{pmatrix} I_i \\ J_i \end{pmatrix} = -\int_0^\infty d\varepsilon F'(\varepsilon) \begin{pmatrix} \tilde{G}_{ij}(\varepsilon) & \tilde{B}_{ij}(\varepsilon) \\ \tilde{\Gamma}_{ij}(\varepsilon) & \tilde{\Xi}_{ij}(\varepsilon) \end{pmatrix} \begin{pmatrix} V_j - V_0 \\ T_j - T \end{pmatrix}, (1)$$

with the derivative $F'(\varepsilon)$ of the Fermi function and the base temperature T. Since there is no net current into the S loop, its potential, V_0, is tuned to ensure $\sum_j I_j = 0$. In two-terminal geometries, Fig. 1a-c, Eq. (1) reduces to $I = G(V_L - V_R) + B(T_L - T_R)$ and $J = \Gamma(V_L - V_R) + \Xi(T_L - T_R)$.

Taking $T_{ij}^{\alpha\beta}(\varepsilon)$ as the transmission coefficient for a β-quasiparticle (e or h) injected from lead j at energy ε exiting as an α-quasiparticle in lead i, Ref. [17] gives

$$\tilde{G}_{ij}(\varepsilon) = \frac{2e^2}{h} \left[2N_i \delta_{ij} - T_{ij}^{ee} + T_{ij}^{he} + T_{ij}^{eh} - T_{ij}^{hh} \right], \quad (2a)$$

$$\tilde{\Xi}_{ij}(\varepsilon) = \frac{2\varepsilon^2}{hT} \left[2N_i \delta_{ij} - T_{ij}^{ee} - T_{ij}^{he} - T_{ij}^{eh} - T_{ij}^{hh} \right], \quad (2b)$$

$$\tilde{B}_{ij}(\varepsilon) = -\frac{2e\varepsilon}{hT} \left[T_{ij}^{ee} - T_{ij}^{he} + T_{ij}^{eh} - T_{ij}^{hh} \right], \quad (2c)$$

$$\tilde{\Gamma}_{ij}(\varepsilon) = -\frac{2e\varepsilon}{h} \left[T_{ij}^{ee} + T_{ij}^{he} - T_{ij}^{eh} - T_{ij}^{hh} \right]. \quad (2d)$$

Average diagonal thermoelectric coefficients. To evaluate the transmission probabilities for Eqs. (2), we use the Feynman rules in Ref. [22], taking into account the presence of two S contacts with phase difference ϕ. We work perturbatively in the ratio N_S/N of the total number of channels carried by the S contacts to those carried by the normal leads. All contributions up to $\mathcal{O}[(N_S/N)^2]$ are shown in Fig. 2, the ϕ-dependent ones are ee2II, he2I, he2II and he2III. For model a and b, their value in Eqs. (3) and (4) of Ref. [22] is now multiplied by $g(\phi) = [N_{SL}^2 + N_{SR}^2 + 2N_{SL}N_{SR}\cos\phi]/N_S^2$. For $N_L = N_R = N$, the average charge and thermal conductances (neglecting weak-localization) are

$$\langle G \rangle = N/2 + N_S^2 g(\phi) f_G(T)/8N, \quad (3a)$$

$$\langle \Xi \rangle = \frac{\pi^2 k_B^2 T}{3e^2} (N/2 - N_S^2 g(\phi) f_\Xi(T)/8N). \quad (3b)$$

The main difference between charge and heat conductances in this symmetric configuration is that the backscattering contribution he2I in T_{LL}^{he} is absent in T_{RL}^{he}, which brings the periodic oscillations in G and Ξ out of phase by π, with G being minimal at $\phi = 0$. This fits with the experiment of Ref. [11]. The thermal dampings are polynomial, generalized zeta and polygamma functions of $\alpha = 4k_B T\tau_D$, where τ_D is the dwell time. Asymptotically $f_G(0) = 1$, $f_\Xi(\alpha \ll 1) \approx 10.4\alpha$, and $f_G(\alpha) \approx \pi/\alpha$, $f_\Xi(\alpha) \approx 32/\pi^2\alpha$, for $\alpha \gg 1$.

Eqs. (3) imply that there are coherent oscillations of the Wiedemann-Franz (WF) ratio

$$\frac{\Xi + \Gamma S}{GT} \simeq \frac{\Xi}{GT} = l_0 \left(1 - \frac{N_S^2 g(\phi)[f_G(T) + f_\Xi(T)]}{4N^2} \right), (4)$$

where our results below show that ΓS is small enough to neglect, and $l_0 = \pi^2 k_B^2/3e^2$ is the Lorenz number. Thus, unless $\phi = \pi$ and $N_{SL} = N_{SR}$ simultaneously, superconductivity causes a $\mathcal{O}[(N_S/N)^2]$ violation of the WF law, parametrically larger than the $\mathcal{O}[N^{-1}]$ violation induced by mesoscopic fluctuations in metallic samples [23].

Average thermopower. The off-diagonal thermoelectric coefficients satisfy the Onsager relation $B = -\Gamma/T$. We checked that our theory preserves this symmetry and only discuss B from now on. In two-terminal arrangements, G is symmetric in ϕ. Therefore the symmetry of the thermopower coefficient $S = -B/G$ is determined by the symmetry of B. To leading order in N_i, particle hole symmetry, $\varepsilon \to -\varepsilon$, is equivalent to reversing the superconducting phases, $\phi \to -\phi$. Combining this with Eq. (2c), we straightforwardly conclude that $\langle B \rangle$ is generically odd in ϕ, up to weak-localization corrections. In a house geometry, however, one can interchange the superconducting leads, and thus reverse the superconducting phases without changing the physics. Thus $\langle B \rangle_a$ must be even in ϕ. Neglecting weak localization corrections, one thus has $\langle B \rangle = 0$ and $S = 0$ for model a.

How can a finite leading-order thermopower emerge? Our symmetry argument breaks down when there are correlations between the action phase a quasiparticle accumulates on its way through the system and the superconducting phase that it picks at Andreev reflections.
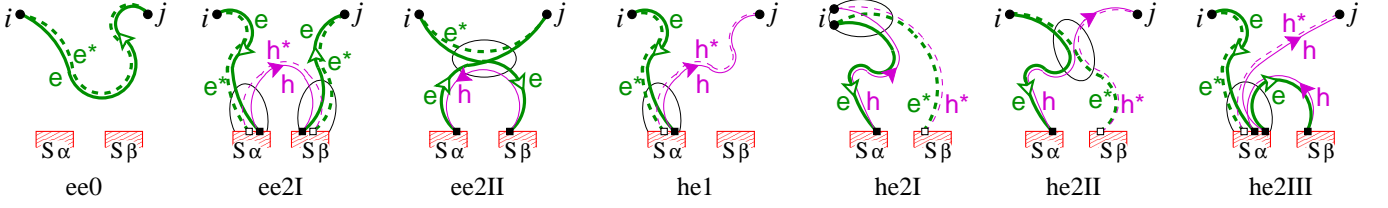
Figure 2: (Color online) Contributions to $\langle T_{ij}^{\mathrm{ee}} \rangle$ (first three) and $\langle T_{ij}^{\mathrm{he}} \rangle$ (last four). Green (violet) paths indicate electrons (holes), dashed lines indicate complex-conjugated amplitudes and circles show trajectory encounters. Normal leads are labelled i, j while superconductors are labelled $S\alpha, S\beta$. Contributions to $\langle T_{ij}^{\mathrm{hh}} \rangle$ and $\langle T_{ij}^{\mathrm{eh}} \rangle$ have e and h interchanged everywhere.

We find that, when present, these correlations generate a finite, odd average thermopower. This is most easily seen by analyzing the asymmetric house interferometer of Fig. 1b, where a neck renders the journey toward SR systematically longer. For simplicity, we assume that all trajectories going into the neck spend a time $\delta\tau$ in it before they hit the SR lead. Now take contribution he2I. If the solid path hits SR and the dashed hits SL, it induces a phase of ϕ from the S leads, and a phase of $2\varepsilon\delta\tau$ from the extra length of the solid path. If the solid path goes to SL and the dashed to SR, we get the opposite phases. Interchanging e and h, means $\phi \to -\phi$, thus these contributions to $T_{ij}^{he} - T_{ij}^{eh}$ behave like $\cos(2\varepsilon\delta\tau + \phi) - \cos(2\varepsilon\delta\tau - \phi)$. The prefactor on this contribution is easily found using the Feynman rules in Ref. [22]. Treating the other contributions in the same way, we find that the leading-order average thermopower for model b is

$$\langle S_{\mathrm{b}} \rangle = -\frac{\langle B_{\mathrm{b}} \rangle}{\langle G_{\mathrm{b}} \rangle} = \frac{4k_{\mathrm{B}}}{e} \frac{N_{\mathrm{SL}} N_{\mathrm{SR}}}{(N_{\mathrm{L}} + N_{\mathrm{R}})^2} \; I_{\mathrm{b}}(T) \; \sin\phi, \quad (5)$$

where $I_{\mathrm{b}}(T) = -(k_{\mathrm{B}}T)^{-1} \int_0^\infty \mathrm{d}\varepsilon \, F'(\varepsilon) \sin(2\varepsilon\delta\tau)/(1 + 4\varepsilon^2\tau_{\mathrm{D}}^2)$. This energy integral is zero for $\delta\tau = 0$ (symmetric house), is linear in T for $T \ll \delta\tau^{-1}, \tau_{\mathrm{D}}^{-1}$, is maximal when $k_{\mathrm{B}}T \sim \delta\tau^{-1} \sim \tau_{\mathrm{D}}^{-1}$ and decays as T^{-1} when $T \gg \tau_{\mathrm{D}}^{-1}$ for $\tau_{\mathrm{D}} \gg \delta\tau$, and as $\exp[-2\pi k_{\mathrm{B}}T\delta\tau]$ for $T \gg \delta\tau$ for $\delta\tau \gg \tau_{\mathrm{D}}$. The average thermopower is always odd in ϕ, but we stress that for it to be finite we need a systematic asymmetry in the distributions of path lengths to SL and SR. An asymmetry in the probability of hitting the two S-contacts, such as for $N_{\mathrm{SL}} \neq N_{\mathrm{SR}}$, is not sufficient.

The presence of the neck with N_{n} channels, in the parallelogram interferometer of Fig. 1c also breaks symmetry between the length of paths to SL and paths to SR. We consider $N_{\mathrm{n}} \ll N$, and treat the problem to leading order in N_{n}/N. The two cavities are not symmetric, and might have different dwell times τ_{DL} and τ_{DR}. To leading order in N_{n}/N and N_S/N we obtain

$$\langle S_{\mathrm{c}} \rangle = -\frac{2k_{\mathrm{B}}}{e} \frac{N_{\mathrm{SL}} N_{\mathrm{SR}} N_{\mathrm{n}} (N_{\mathrm{SL}} - N_{\mathrm{SR}}) \, I_{\mathrm{c}}(T) \, \sin\phi}{N_{\mathrm{L}} N_{\mathrm{R}} \left[N_{\mathrm{n}}(N_{\mathrm{SL}} + N_{\mathrm{SR}}) + 2N_{\mathrm{SL}} N_{\mathrm{SR}} \right]}, \quad (6)$$

where $I_{\mathrm{c}}(T) = -(k_{\mathrm{B}}T)^{-1} \int_0^\infty \mathrm{d}\varepsilon \, \varepsilon \, F'(\varepsilon) \, 2\mathrm{Im}\left[\mathrm{e}^{\mathrm{i}2\varepsilon\delta\tau}/A(\varepsilon) \right]$ with $A(\varepsilon) = (1 + \mathrm{i}2\varepsilon\tau_{\mathrm{DL}})(1 + \mathrm{i}2\varepsilon\tau_{\mathrm{DR}})$. Again the thermopower is odd in ϕ, but to be finite it requires $N_{\mathrm{SL}} \neq$

N_{SR}. In fact, for $N_{\mathrm{SL}} = N_{\mathrm{SR}}$ there are odd-ϕ contributions to $\langle S_{\mathrm{c}} \rangle$ at next order in $N_{\mathrm{S},n}/N$ (assuming $\tau_{\mathrm{DL}} \neq \tau_{\mathrm{DR}}$). We note in passing that a similar expression is obtained for the hook geometry of Ref. [10].

We apply this (N_{n}/N)-perturbation theory to the three-dot model of Fig. 1d. The L and R leads have voltages $V_{\mathrm{L}}, V_{\mathrm{R}}$ such that no current flows in any lead (or S contact) when lead M is held at a temperature $T + \delta T$. The thermopower $S_{\mathrm{d}}^{\alpha\mathrm{M}} = V_\alpha/\delta T$, $\alpha \in \mathrm{L}, \mathrm{R}$, is lead-dependent. To find this, we solve Eq. (1) for V_0, V_{L} and V_{R} when all charge currents are zero. One obtains

$$\langle S_{\mathrm{d}}^{\mathrm{LM}} \rangle = \frac{2k_{\mathrm{B}}}{e} \frac{N_{\mathrm{nR}}^2 N_{\mathrm{nL}} N_{\mathrm{SL}} N_{\mathrm{SR}} \, I_{\mathrm{d}}(T) \, \sin\phi}{N_{\mathrm{M}} N_{\mathrm{L}} N_{\mathrm{R}} \left[N_{\mathrm{nL}} N_{\mathrm{nR}} + N_S'(N_{\mathrm{nL}} + N_{\mathrm{nR}}) \right]}, \quad (7)$$

where $N_S' = 2N_{\mathrm{SL}} N_{\mathrm{SR}}/(N_{\mathrm{SL}} + N_{\mathrm{SR}})$, and $I_{\mathrm{d}}(T)$ is given by $I_{\mathrm{c}}(T)$ with $\delta\tau \to (\delta\tau_{\mathrm{L}} + \delta\tau_{\mathrm{R}})$ and $A(\varepsilon)$ gaining a factor of $(1 + 2\mathrm{i}\varepsilon\tau_{\mathrm{DM}})$. Similarly we find $\langle S_{\mathrm{d}}^{\mathrm{RM}} \rangle = -(N_{\mathrm{nL}}/N_{\mathrm{nR}})\langle S_{\mathrm{d}}^{\mathrm{LM}} \rangle$. Thus for $N_{\mathrm{nR}} \simeq N_{\mathrm{nL}}$, one obtains $\langle S_{\mathrm{d}}^{\mathrm{RM}} \rangle \simeq -\langle S_{\mathrm{d}}^{\mathrm{LM}} \rangle$. All this fits with the experimental data or Ref. [13].

It is worth recalling that Eqs. (5–7) are leading order in N_{S}/N, and thus neglect oscillations $\propto N_{\mathrm{S}} \cos\phi/N$ in the denominator of $\langle S \rangle$ (coming from $\langle G \rangle$). These terms generate higher odd-ϕ harmonics, not unlike the experimental findings.

Mesoscopic fluctuations. Since the average thermopower vanishes for model a, we look at mesoscopic fluctuations. We consider contributions to $(T_{\mathrm{RL}}^{ee} - T_{\mathrm{RL}}^{hh} + T_{\mathrm{RL}}^{he} - T_{\mathrm{RL}}^{eh})$ before mesoscopic average. The diagrams shown in Fig. 3 give a contribution $(T_{\mathrm{RL}}^{ee} - T_{\mathrm{RL}}^{hh}) \propto \sin[\varepsilon(T - t_2)] \sin[\mathrm{i}2E_{\mathrm{F}}(T + t_2) + \phi]$ for $T = t_1 + t_3 - t_4$. We sum over the 24 permutations of the four trajectory durations t_i, $i = 1, 2, 3, 4$, which generally gives both even-ϕ and odd-ϕ contributions. For the special case of the symmetric house, however, the odd contributions cancel out exactly, since for every contribution touching both superconducting contacts, there is a contribution with equal weight touching the superconducting contacts in the reversed sequence. These two come with opposite signs of ϕ, so the sum is even in ϕ. An analysis of $T_{\mathrm{RL}}^{he} - T_{\mathrm{RL}}^{eh}$ shows the same behavior, therefore the sample-dependent thermopower must be even in ϕ in this case.

To evaluate the typical magnitude S_{typ} of the thermopower in a single measurement, we estimate for $\mathrm{var} B_a$.
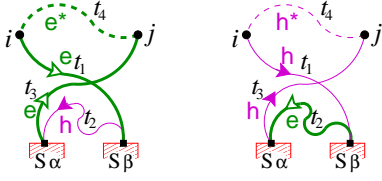
Figure 3: Contributions to ee2II and hh2II before ensemble averaging.

This is done by pairing any two contributions in Fig. 2, which necessitates to add at least two encounters. From the Feynman rules in Ref. [22], one obtains the leading order in N_S/N_T by pairing ee0 with ee0, ee2II and he2III, including all permutations with e$\leftrightarrow$h. Then $B_{\rm typ} \approx$ rms$[B_a] \sim (\pi^2 k_{\rm B}^2 T/3e)\tau_{\rm D}[\kappa_1 + \kappa_2 N_{\rm SL}N_{\rm SR}\cos(\phi)/N^2]$ at low temperature $k_{\rm B}T\tau_{\rm D} \ll 1$ where $\kappa_{1,2} = \mathcal{O}[1]$. The first term is dominated by normal metal fluctuations of B [24]. In contrast to Aharonov-Bohm oscillations, mesoscopic fluctuations of thermoelectric coefficients are not parametrically enhanced by the presence of superconductivity. A single measurement of the house interferometer of Ref. [10] thus typically produces an even$-\phi$ thermopower,

$$S_{\rm typ} \approx (2k_{\rm B}/e)\big(S_0 + S_1 N_{\rm SL}N_{\rm SR}\cos(\phi)/N^2\big), \quad (8)$$

which need not vanish at $\phi = 0$. The constants $S_{0,1} \sim (2e^2/h)G^{-1} \ll 1$, so the above $\cos\phi$-term is in principle smaller than the odd thermopowers, Eqs. (5–7). For a given sample, $S_{0,1}$ are random in sign. In a symmetric house geometry, Ref. [10] reported an even thermopower at $T = 38$mK of similar magnitude as the odd thermopower found in a parallelogram geometry at $T = 350$mK. This might be due to a much stronger temperature damping in the latter case, or to $N_{\rm SL} \approx N_{\rm SR}$ in the parallelogram, or both. We note that the experiments found $S_{\rm a}(\phi = 0) \neq 0$, in agreement with our theory.

Conclusions. Our theory potentially explains all existing thermopower experiments on Andreev interferometers [5, 6, 10, 11, 12, 13]. Finite average thermopowers are systematically odd in ϕ, and emerge when the geometry correlates trajectory durations to superconducting contacts with the phase at the contacts. When the average thermopower vanishes, mesoscopic fluctuations cannot be neglected. We discussed the latter for the first time and showed that they are systematically even in ϕ. Unlike earlier theories [18, 20, 21], our mechanisms for thermopower do not presuppose charge imbalance nor require temperature differences between the superconducting contacts. Waiving the latter requirement allows us in particular to explain the odd thermopower recently found in hot-middle interferometers [13]. We hope that the validity of our theory for even S in house interferometers will soon be checked by investigations of mesoscopic fluctuations.

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